Asymptotic Properties of Discrete and Picewise Models of Additive Increase Multiplicative Decrease Algorithm.

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Abstract Random walks with additive increase and multiplicative decrease are widely used for performance control and modeling in telecommunication, smart spaces and some biological systems as well. There exists in the literature two mainstream approaches which apply discrete stepwise and piecewise linear random processes. Meanwhile most real implementations of the algorithms used by the networking applications support discrete arithmetics for its key variables. Therefore piecewise linear models provide approximate results and the applicability of these results needs further studies. In the paper we consider the connection between discrete stepwise and piecewise linear models and provide the boundary estimation for the important characteristic of the stepwise random process in terms of the piecewise linear random process.

Keywords: Stochastic analysis, Random walk, Data communication, AIMD

1 Introduction

Random walks with additive increase and multiplicative decrease are widely used in the modern networking environments for distributed control of the communication parties activity [19,22,24,25,23,20,21,9,7,14,8,17,16]. The Additive Increase Multiplicative Decrease (AIMD) algorithm implements the random walk to provide flow control at the Internet transport layer. Different variations of the algorithm are used by more than ten Transmission Control Protocol (TCP) protocol implementations. According to AIMD, a source increases sending rate if there is end-to-end route capacity available and decreases the rate if it receives a congestion signal. Congestion avoidance AIMD algorithm described by [11], [2], i.e. New Reno TCP version is widely implemented. The algorithm disadvantages on the end-to-end paths that include high-speed, high bandwidth delay product value or wireless links are widely discussed [4], [1]. Nevertheless the algorithm provided exponential growth of the Internet during more than twenty years. Also its performance is used as a measure of fair share of the networking infrastructure and for tuning parameters of the experimental TCP versions, see e.g. [13]. Thus better understanding of New Reno behavior provides basis for further research and a tool for an administrative solutions for the different networking environments as well. At present, NewReno version of TCP is implemented in a wide variety of modern OS kernels' networking modules.

More sophisticated variations of the algorithm based on AIMD-kind random walks are proposed to provide distributed performance control for highly congested publish/subscribe IoT environments and smart spaces [26], [15]. As for the data communication networks the algorithms are used by data sources so in IoT environments or smart spaces they are implemented by the clients subscribed for Semantic Information Brokers (SIB) service notifications. The clients control the periods between notification requests. They increase it linearly if no losses of notifications happened and decrease it by multiplication factor if the losses occurred. The factor depends on the number of losses and several other arguments.

Wide scope of the applications and strict demands to their performance define the importance of modeling and analysis studies of the significant properties of the random walks mentioned above.

In many cases their key performance metrics could be described by stepwise random process with semi-markovian or renewal properties [12]. The state space of the process is the set of non-negative integers and multiplication is followed by the floor operation due to the nature of the communication protocols i.e. amount of data expressed in bytes, number of rounds etc. are measured in discrete values. Hence the corresponding random variables follow discrete probability distributions as well. Nevertheless in most researches [1] the stepwise process is substituted by piecewise random process [12] with polynomial (as usual linear) growth periods with markovain or renewal properties as well and the floor operation is neglected. Therefore the space of state of the piecewise linear random process is formed by non-negative real numbers. The piecewise linear models allow avoiding many analytical problems rose by stepwise models, make models simpler and tractable, but they ignore discrete nature of the applications, since the real values provided by AIMD variables are rounded before further processing of the data to send. The substitute allows using of the powerful methods of continuous functions analysis and hence yields simpler models and stronger results. Meanwhile there are few works those research a connection between the stepwise and corresponding piecewise linear process. In the paper we study connection between parameters of such processes and their asymptotic behavior.

There exists in the literature two different approaches to the description of TCP data loss process. The first one considers the losses as a random flow. So if τ_k and τ_{k+1} are the moments of two consequent data losses, then the type of distribution of $[\tau_{k+1} - \tau_k]$ intervals is an essential assumption of the model, see e.g. [3]. The second approach treats the sequence of data sent and demands the definition of the distribution function of r.v. S_n which is amount of data sent during $[\tau_{k+1} - \tau_k]$ interval, e.g. [18]. Generally the area of TCP behavior research and modeling is rather large, therefore for further information about state-of-art in the area one can address to the survey [1].

This work considers the random walk implemented by AIMD algorithm as stepwise process and piecewise linear process and establishes boundary estimate which bounds steady state second moments of the congestion window size provided by these two models. To define data loss process we consider the sequence of data sent between two consequent loss events. Following most of publication we consider loss event as it is defined by NewReno TCP version and evaluate the goodput.

The work is organized as follows. Section 2 describes two baseline models of the AIMD random walk, Section 3 presents the theorem about the boundary estimation and its proof. Section 4 contains conclusions.

2 Two baseline models of the additive increase and multiplicative decrease random walk

First we describe the stepwise model of the random walk. Let us define the random process which describes a behavior of AIMD congestion window size in the terms of TCP segments.

Let t_n be the moments of AIMD-rounds end-points. Hence $[t_{n-1}, t_n]$ are round trip intervals, and RTT length is $\xi_n = t_n - t_{n-1}$. Let us denote $w(t)$ a congestion windows size under AIMD control at the time moment t. Then $\{w(t)\}_{t>0}$ is a stepwise process such that

$$w(t_n + 0) = \begin{cases} \lfloor \alpha w(t_n) \rfloor, & \text{if during the interval (TCP round) } [t_{n-1}, t_n] \\ & \text{one or more TCP segment losses has happened,} \\ w(t_n) + 1, & \text{if all data in the round were successfully delivered.} \end{cases}$$

Between the moments t_n the process $\{w(t)\}_{t>0}$ stays constant and $\alpha < 1$.

Let us suppose that ξ_n are independent identically distributed (iid.) random variables with distribution $R(x)$, which is absolutely continuous on the set $\mathbb{R}^+$ and $\mathsf{E}[\xi_n] < \infty$. Let us denote sequence $\{S_n\}_{n>0}$ which describes amounts of data sent between two consecutive loss events. We assume that S_n are iid. random variables with finite expectation $\mathsf{E}[S_n] < \infty$. The count of data sent starts from the round next to the loss event. Let us denote $w_n = w(t_n)$ and let $\tau_k = t_n$ if a loss event happened during ξ_n period, i.e.

$$w(\tau_k + 0) = \lfloor \alpha w(\tau_k) \rfloor .$$

Also let us denote $W_k = w(\tau_k)$ so $W_k = w_n$ if $\tau_k = t_n$.

Then the sequence $\{W_k\}_{k>0}$ forms the Markov chain embedded in the random process $\{w(t)\}_{t>0}$. Fig. 1 presents graphical example of $\{w(t)\}_{t>0}$ evolution.

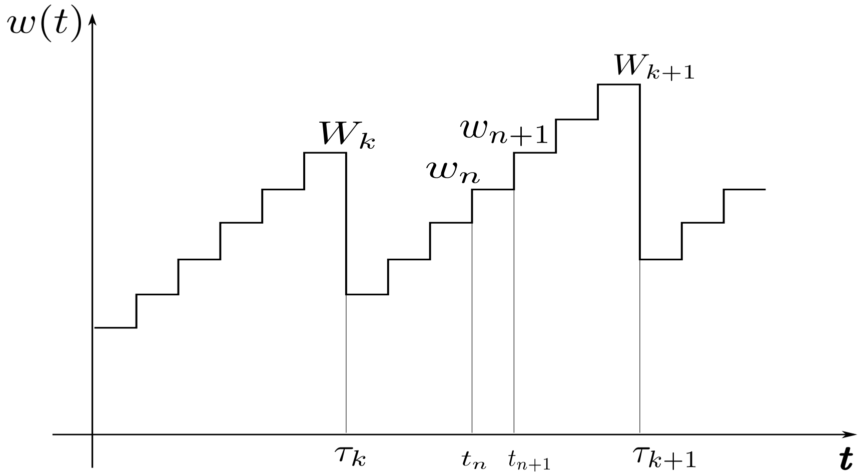

Fig. 1. Stepwise random process

Now we consider a piecewise linear random process which presents the evolution of the random walk. Let $\{X(t)\}_{t>0}$ take values from $\mathbb{R}^+$ growing linearly in the intervals $[\theta_n, \theta_{n+1})$ $n = 0, 1, \ldots$ with the speed $b = \mathsf{E}[\xi_n]^{-1}$, i.e. $X(t) = X(t_0) + bt$, $\forall$ $[t_0, t] \subset [\theta_n, \theta_{n+1})$. At random moments $\{\theta_n\}_{n\geq 0}$ the process $\{X(t)\}_{t>0}$ makes jumps $X(\theta_n + 0) = \alpha X(\theta_n)$, where $\alpha < 1$ and $X(\theta_n - 0) = X(t_0) + b\theta_n \neq \alpha X(\theta_n)$.

We assume that amounts of data sent between the moments $\{\theta_n\}_{n\geq 0}$ define a sequence $\{\eta_n\}_{n>0}$ which forms renewal process with continuous density renewal function, cumulative distribution function $F(x)$ and $\mathsf{E}[\eta_n] < \infty$. We introduce a sequence $\{X_n\}_{n>0}$ such that $X_n = X(\theta_n)$, thus $X_n = X(t)$ if $X(t+0) = \alpha X(t)$. So a multiplicative decrease happens after each X_n value. Let us notice that the sequence $\{X_n\}_{n\geq 0}$ possesses Markovian property and it is embedded in the process $\{X(t)\}_{t>0}$. Fig. 2 presents graphical example of $\{X(t)\}_{t>0}$ evolution.

There are many publication which derive $\mathsf{E}[W_n]$ or $\mathsf{E}[X_n]$ estimates under various conditions and assumptions. The following section studies relation between these two values.

3 Comparison of the models

Under assumptions formulated above the following relation between stationary (equipped with asterisk) expectations $\mathsf{E}[{X^*}^2]$ and $\mathsf{E}[{W^*}^2]$ holds

Theorem 1. *If* $\mathsf{E}[S_n] = \mathsf{E}[\eta_n]$ *and* $b = 1$ *then*

$$\mathsf{E}[{X^*}^2] - \frac{1}{1-\alpha^2} \leq \mathsf{E}[{W^*}^2] \leq \mathsf{E}[{X^*}^2].$$

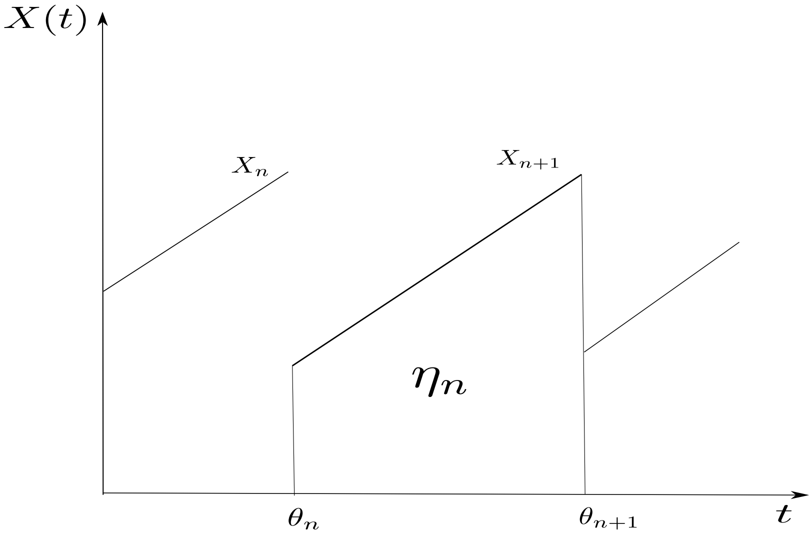

Fig. 2. The piecewise linear random process of the congestion window size.

Proof. Simple geometrical considerations provide the following equation for the sequence W_n^2 in the following form [10]

$$W_{n+1}^2 - \alpha^2 W_n^2 = \frac{2}{p},$$

where p is data segment loss probability for Bernoulli loss process. Using assumptions made on the piecewise process and following heuristic approach presented in [10] yields

$$W_{n+1}^2 = \lfloor \alpha^2 W_n^2 \rfloor + 2S_n, \tag{1}$$

where $\lfloor x \rfloor$ is the largest integer not exceeding x. Discrete nature of the units and calculation methods used by networking software determine the using of the floor operation in the equation (1). Nevertheless the operation poses significant difficulties for the further analysis. Therefore let us transform the equation (1) into the following form

$$W_{n+1}^2 = \alpha^2 W_n^2 - \gamma_n + 2S_n,$$

where γ_n is the random value and $0 \leq \gamma_n \leq 1$. According to [5] the Markov chain $\{W_n\}$ converges to steady state distribution if $\mathsf{E}[S_n]$ is finite. Let us study heuristically the following dynamic

$$\tilde{W}_{n+1}^2 = \alpha^2 \tilde{W}_n^2 + (2S_n - \gamma_n). \tag{2}$$

Applying recurrent equation (2) one can obtain stationary solution

$$W_n^{*^2} = \sum_{i=0}^{\infty} \alpha^{2i} [2S_{n-i-1} - \gamma_{n-i-1}].$$

Therefore one can calculate the corresponding expectations as follows

$$\mathsf{E}[W_n^{*^2}] = \sum_{i=0}^{\infty} \alpha^{2i} \mathsf{E}[2S_{n-i-1} - \gamma_{n-i-1}]$$

and, hence

$$\mathsf{E}[W_n^{*^2}] = \sum_{i=0}^{\infty} \alpha^{2i} \left(\mathsf{E}[2S_{n-i-1}] - \mathsf{E}[\gamma_{n-i-1}]\right)$$

Notice that since $\alpha < 1$ and $\forall\ n$ $\mathsf{E}[S_n]$ and $\mathsf{E}[\gamma_n]$ are finite the latter series converges absolutely and therefore

$$\mathsf{E}[W_n^{*^2}] = \frac{1}{1-\alpha^2} \left(2\mathsf{E}[S_n] - \mathsf{E}[\gamma_n]\right). \tag{3}$$

Since $0 \leq \gamma_n \leq 1$ then $0 \leq \mathsf{E}[\gamma_n] \leq 1$ as well, and thus

$$\frac{2}{1-\alpha^2}\mathsf{E}[S_n] - \frac{1}{1-\alpha^2} \leq \mathsf{E}[W_n^2] \leq \frac{2}{1-\alpha^2}\mathsf{E}[S_n].$$

Now let us consider the process $X(t)$. According to [3] the sequence X_n can be obtained from the following system

$$(X_{n+1} + \alpha X_n)(X_{n+1} - \alpha X_n)\frac{1}{2b} = \eta_n.$$

Then after trivial transformation one obtains

$$X_{n+1}^2 = \alpha^2 X_n^2 + 2b\eta_n. \tag{4}$$

Stochastic equation (4) satisfies conditions of convergence theorem formulated in [6] since $\alpha < 1$ and $\mathsf{E}[\eta_n] < \infty$ and therefore it has the only finite stationary solution in the following form

$$X_n^{*^2} = \sum_{i=0}^{\infty} \alpha^{2i} \left(2b\eta_{n-i-1}\right)$$

Therefore applying expectation one obtains

$$\mathsf{E}[X_n^{*^2}] = 2b \sum_{i=0}^{\infty} \alpha^{2i} \mathsf{E}[\eta_{n-i-1}]$$

and hence

$$\mathsf{E}[X_n^{*^2}] = \frac{2b}{1-\alpha^2}\mathsf{E}[\eta_n]. \tag{5}$$

Setting according to the theorem condition $\mathsf{E}[\eta_n] = \mathsf{E}[S_n]$ and $b = 1$ one can see that first term in the right hand side of (3) is equal to the expression in the right-hand side of (5)

$$\mathsf{E}[X^{*^2}] - \frac{1}{1-\alpha^2} \leq \mathsf{E}[W^{*^2}] \leq \mathsf{E}[X^{*^2}]$$

which proves the theorem. ◻

Let notice that e.g., NewReno version uses value $\alpha = 1/2$ and congestion window size normally fluctuates from several tens to several hundreds of segments for wide range of applications. In the case according to the theorem the error of the piecewise model will be smaller than 4/3 which is insignificant for the practical purposes.

Now let us consider the parameter b of the $X(t)$ process and its role in the modeling and estimates evaluation. The discrete stepwise models [10], [18] and many others consider congestion window size evaluation in the discrete time scale reduced to the natural numbers. Thus the embedded markov chain addresses to the number of AIMD round and does not consider its length ξ_n. Hence congestion window size becomes independent on RTT duration. This assumption fairly reflects features of many practical networking environments and end-to-end paths where TCP segment loss probability does not depend on the RTT duration. Moreover very reliable end-to-end paths may have long RTT periods e.g. those incorporating satellite channels. Nevertheless RTT is used in calculation of the throughput since there it could not be discarded. Thus the throughput is estimated as

$$B_n = \frac{S_n}{T_n},$$

where

$$T_n = \sum_{k=1}^{m_n} \xi_k.$$

Here m_n is the number of rounds which the sender needs to reach congestion window size W_{n+1} starting from congestion window size $\lfloor \alpha W_n \rfloor$. Average m_n is estimated by most researchers as $(1-\alpha)\mathsf{E}[W_n]$ [18], [1], [16], [14].

For the piecewise model, parameter b describes linear growth rate and depends on two parameters of the networking environment. These are congestion window increment size δ and RTT duration. These models use the ratio

$$b = \frac{\delta}{\mathsf{E}[\xi]},$$

see [13],[3], [8], [1]. Therefore, piecewise linear models, exploring continuous time, take into account RTT influence on congestion window size through ratio b, which is used in the estimations of $\mathsf{E}[X_n]$. So stepwise model considers more narrow set of arguments for the evaluation of $\mathsf{E}[W_n]$, than the picewise models do evaluating $\mathsf{E}[X_n]$. Nevertheless both models use same set of the arguments for the throughput (goodput) estimates. The theorem proved above uses the restriction $\delta = 1$ since it does not reduce the generality of the result.

4 Conclusion

In this paper the connection between stepwise and piecewise models of additive increase multiplicative decrease random walk is obtained. The random walk describes a behavior of the networking software critical algorithms, smart spaces applications and some biological systems. The Markov chain embedded in the stepwise random process and Markov sequence embedded in the piecewise linear random processes are considered, and the theorem relating their parameters and characteristics is proved. The results obtained demonstrate that piecewise linear model produces good estimation of the performance metrics for the discrete algorithms based on the additive increase multiplicative decrease random walk.

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